

Structural State Identification during Vertical Rotation Construction of a Bridge Based on Lightweight Monitoring

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Abstract: To address the uncertainty of structural states during construction and the scarcity of field monitoring data, which make it difficult to fully characterize structural pose and force conditions, we developed a state identification method based on limited monitoring information. For the reference state, the key control parameters governing the equilibrium of the initial configuration were identified. For the construction process states, pose-correction parameters and key state parameters were jointly identified from a small number of observations. State identification was implemented by integrating finite element analysis with a global-local optimization strategy. The vertical rotation construction of the main arch of an overpass bridge was investigated as an engineering case. The results show that the method can accurately recover the prescribed cable force configuration in the pre-construction reference state and stably identify spatial pose changes and cable force redistribution at several discrete construction states. The error indices remained generally small, and the results at the constraint points agreed well with those at the validation points. The

proposed method provides an effective technical approach for state assessment, geometric control, and parameter updating during the construction of complex bridges.

Keywords: structural state identification; limited monitoring information; pose correction; cable force identification; finite element analysis

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1 Introduction

The development of bridge construction-control technology in China has progressed from empirical adjustment to computer-aided analysis and state identification. Since the 1980s, the increasing application of computer technology in engineering has gradually shifted bridge construction control from conventional judgment based on experience to parameter adjustment and state correction based on numerical analysis [1]. Early engineering practices represented by Liugang Bridge [2] marked the beginning of the application of modern engineering control theory to bridge construction in China. These practices also promoted the extension of bridge construction monitoring from a single bridge type to cable-stayed bridges, suspension bridges, arch bridges, continuous rigid-frame bridges, and other bridge types. On this basis, construction control has evolved from simple geometric alignment adjustment to an integrated control problem that considers structural forces, the construction pose, and stage-specific state assessment.

The use of optimization-based inverse algorithms that combine neural networks and finite element analysis for structural parameter estimation and identification in real-world contexts has demonstrated the feasibility of structural state identification [3]. State identification during construction is important because the completed alignment and internal force state of a long-span bridge are closely related to its

construction process, and the construction method and erection sequence directly affect the final completed state [4-6]. In addition, errors in design parameters accumulate during construction. If these errors are not identified and corrected progressively, structural forces and deformations may deviate from their target states, thus compromising safety and service performance [7]. Nevertheless, obtaining field data is difficult. Field state assessment depends heavily on manual observation, and the collection of construction-state data is time-consuming and labor-intensive [8]. Although subsequently proposed Internet of Things solutions can provide position, identity, and trajectory information, they may interfere with normal construction activities and are affected by noise, deployment, and maintenance issues [9]. Vision-based methods avoid direct contact, but their application at complex construction sites is limited by occlusion, camera vibration, and inadequate real-time performance and automatic tracking in multi-person scenarios [8].

Accordingly, effective identification of the current structural state from limited observations under continuously changing construction conditions and incomplete field information is a key problem in construction control. A parameterized state description is therefore introduced to represent structural geometry and force conditions using a finite set of parameters. A numerical model is then used to establish mapping between the state parameters and observed responses, thus transforming construction state identification into an inverse parameter problem constrained by limited observations. The resulting method jointly identifies the structural pose and key force-related parameters through optimization.

2 Model of the Structural State-Identification Problem

2.1 Structural State-Identification Scenario

The construction of complex bridges and large civil infrastructure is distinctly stage-dependent. Unlike the stable service state after completion, a structure under construction typically experiences temporary support, segmental assembly, structural-system conversion, and progressive release of boundary restraints. Its load-resisting system, geometry, and constraints thus change continuously with the construction sequence. This behavior is particularly pronounced for long-span structures, spatially curved structures, and projects involving critical operations such as rotation and closure. Between construction stages, the structure does not simply transition from one stable state to another; rather, it undergoes a complex redistribution of forces in which temporary facilities and the permanent structure act together. The structural response is therefore governed not only by the designed completed state but also by the construction sequence, temporary support system, lifting or rotation equipment, cable system adjustment, and environmental factors. Changes in boundary conditions and temporary constraints may alter the geometric state, thus increasing the difficulty of construction control and making it more difficult for the actual construction state to approach the intended state [10].

Accurate identification of the structural state is fundamental to construction process control under these conditions. As construction progresses, the spatial position and force state of the structure vary continuously. Without timely and reliable assessment, errors may accumulate in subsequent operations and adversely affect alignment, structural safety, and construction accuracy. In practice, however, directly available field data are generally incomplete. Because of occlusion, limitations on sensor locations and quantities, environmental interference, and stage-dependent changes in monitoring conditions, only a few key point coordinates, local displacements, or other limited measurements are often available. A continuous and complete observation chain is thus difficult to establish. This is especially true when construction organization is complex and spatial access is restricted: some monitoring devices cannot operate stably over long periods, and measurements may be obtained only discretely under critical conditions. Consequently, direct

reconstruction of the structural state from dense, full-field monitoring is impractical, and the available information is characteristically discrete and local.

The structural state identification problem during construction can thus be abstracted as state identification under limited observations. The objective is not to reproduce every detail of the entire construction process but to reasonably estimate the current overall pose and key force state of the structure from a small amount of measured information. When a complete monitoring chain cannot be established, state parameters can be inferred from limited data through matching between model predictions and observations. Construction control can thereby be transformed from a process based solely on empirical judgment into a computable and verifiable analysis.

2.2 Representation of State Parameters

A structure remains in a continuously changing state during construction, and its geometric configuration, boundary constraints, and loading conditions are adjusted as construction proceeds. Unlike the stable state after completion, the construction state should not be described directly using high-dimensional and dispersed physical quantities. Instead, it should be abstracted into a small number of state parameters that characterize the principal structural responses. These parameters are not limited to a single physical quantity for a specific project type. They summarize the deviation of the structure from the state at a particular construction instant and its corresponding mechanical-response characteristics and may include configuration-correction parameters, boundary-condition correction parameters, and key force-related parameters associated with the construction process.

Under limited observation conditions, directly available field information is generally sparse and may include only a small number of key point coordinates, local displacements, or other measurements. Although few in number, these observations can provide the basic information required for state identification when their locations are representative and reflect the overall structural response.

The state parameters and observed variables do not have a simple one-to-one relationship; instead, they are linked by nonlinear mapping through the structural response. This mapping is not purely geometric fitting but a process involving both the actual geometric configuration and the mechanical state. Changes in observation point positions may result from configuration correction, force redistribution, or altered boundary conditions. Geometric matching alone is thus generally insufficient to accurately describe the construction state. Changes in configuration and force conditions jointly affect the spatial positions of observation points and the values of other monitored quantities. Conversely, differences between the measured and calculated values indicate deviations between the current and target states. On this basis, an optimization model can be formulated to minimize observation errors. The state parameters are iteratively adjusted until the model predictions approach the measurements, thus enabling quantitative identification of the current structural state. The incorporation of the state parameters and observed variables into a unified framework permits the simultaneous consideration of configuration updating and force-state identification and provides a clear mathematical formulation for the subsequent optimization.

2.3 Solution Conditions for State Identification

To formulate the construction-stage state identification problem mathematically and ensure that it can be solved in practice, reasonable abstractions are required for the manner of state variation, observation conditions, and error characteristics during construction. Although construction processes are complex and stage-dependent, the dominant factors within an individual stage are generally concentrated within a small number of key parameters, and the structural response can often be approximated by these parameters. Construction-stage state variation

may thus be treated as a low-dimensional parameter-driven process. Complex spatial changes and force redistribution can be simplified as variations in a finite-dimensional state vector, transforming an originally high-dimensional continuous structural problem into a solvable parameter identification problem.

Field observations are generally local and cannot fully characterize the entire structural state. With an appropriate sensor layout, however, a small number of key observation points can still effectively constrain the overall configuration and principal force characteristics. State identification thus does not depend on dense full-field monitoring but on the assumption that a few representative observations can constrain the dominant state parameters. For the subsequent modeling and solution, the discrepancies between the observations and model predictions are expressed uniformly as error terms and are used to construct the principal objective function. Construction state identification can thus be formulated as a parameter-updating problem constrained by limited observations and governed by error minimization. The solution does not seek to recover every structural detail but focuses on identifying the state parameters most important to construction control while ensuring that the results remain consistent with the construction conditions in both geometric and mechanical terms.

3 Engineering Case and State-Identification Method

3.1 Project Overview

The vertical rotation stage of the main arch construction scheme for an overpass bridge was selected as the engineering case. The vertical rotation was directly controlled by two layers of $\phi 1860$ prestressing-strand cables, as shown in Figure 1. The main bridge is a half-through concrete-filled steel tubular truss arch with a main span of 300 m. It is characterized by a large span, numerous components, finely divided construction stages, and stringent process control requirements. Because the main arch comprises many segments, the spatial pose of each segment must be controlled precisely. Construction organization must ensure segmental assembly accuracy while also satisfying the force safety requirements of the temporary system and controlling the overall alignment. The project thus has a large structural scale, numerous stages, and complex temporary measures typical of long-span bridge construction.

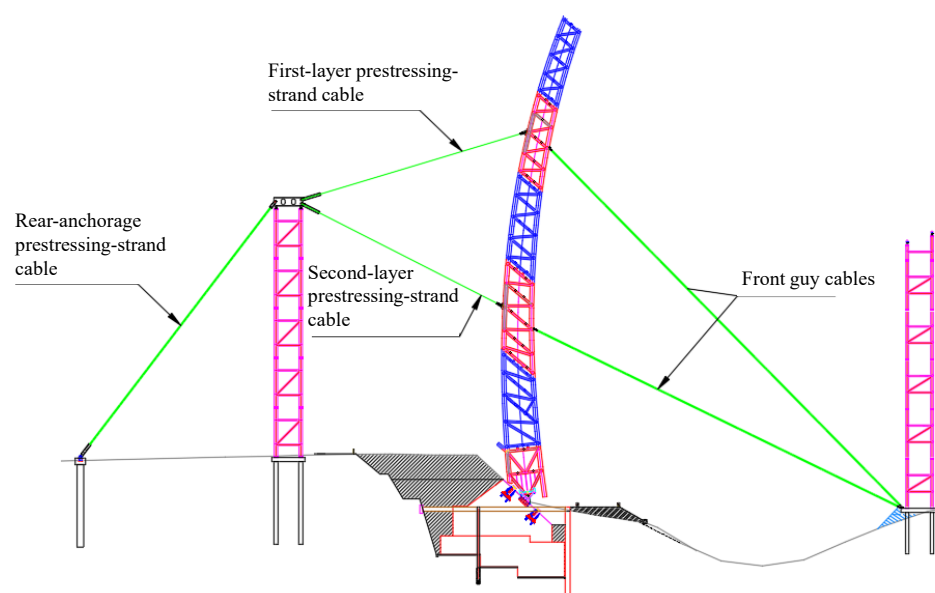


Figure 1 Schematic of the vertical rotation construction of the main arch

As construction progressed, the hinged connections at the arch springing were not fully fixed. During vertical rotation and subsequent operations, the structure progressively deviated from its original vertical plane and exhibited spatial inclination and torsion. This behavior indicates that a structure under construction does not necessarily remain in the ideal force state; instead, its spatial pose may change because of variations in boundary conditions and adjustments to temporary restraints. For such off-design conditions, neither the design state nor visual inspection alone can accurately characterize the actual spatial position and force state. A state-identification method is thus required to quantitatively track and evaluate the structural configuration during construction.

The main-arch construction sequence includes segmental assembly, temporary support, vertical rotation, and concrete placement. Both spatial position and boundary conditions change markedly across stages. Unlike the completed state, the construction state is not fixed but evolves progressively with construction, including gradual configuration adjustment, transformation of the load-resisting system, and continuous changes in spatial pose. These pronounced state changes make the structure a suitable case for verifying a state-identification method under limited observations.

The structural position and pose before and after construction can be characterized using design drawings and spatial measurement point data acquired during construction. Variations among construction stages further reveal changes in structural response. Coordinates measured at different stages can therefore be used as inputs for state identification analysis. By comparing the design and construction states, an identification model for structural pose and force conditions can be established.

3.2 Construction of the State Identification Model

The objective of construction state identification is to determine the spatial configuration and force-related parameters of the structure at a specified construction stage from observable information so that the computational model approximates the actual state as closely as possible. Because the pre-construction reference state and construction process states rely on different sources of information, their identification variables, constraints, and solution procedures also differ. Separate identification models must therefore be established and solved within a unified optimization framework, as summarized in Table 1.

Table 1 Variables and solution procedures for the state identification models

State type	Data sources	Variables to be identified	Principal constraints	Solution characteristics
Pre-construction reference state	Drawing-based model; reference configuration	Cable force parameters	Nodal positions; configuration deviation	Cable force equilibrium
Construction-process state	Measured points; initial model; reference configuration	Pose parameters; cable force parameters	Constraint point errors; validation point errors	Registration and joint optimization

The pre-construction reference state is established primarily from the design drawings and construction control requirements. Identification focuses on the cable force parameters required to maintain the reference configuration. At this stage, the structural geometry is considered known: control point coordinates, component positions, and nodal connectivity are obtained directly from the drawings and reference model, whereas cable forces are the only unknown variables in the optimization. The response of the structure in the reference state is expressed as the equilibrium response under the cable force parameters. By adjusting the initial force in each cable, the calculated nodal displacements are made as close as possible to the reference configuration, thus obtaining the cable force configuration for the reference state. This process is essentially a parameter identification problem that uses the drawing-based geometry as the reference and the cable force equilibrium as the objective. The original geometric information is provided directly by the drawing-based model, and the optimization variables are limited to the cable force parameters.

Construction process states differ from the pre-construction reference state. As construction progresses, the spatial position of the structure changes continuously, and temporary constraints and load boundaries are adjusted between stages. Cable forces alone cannot fully characterize the current state. Overall, pose-correction quantities must thus be introduced in addition to the cable force parameters such that the current construction state is represented through joint identification of the pose and cable force parameters. Pose correction is described by six degrees of freedom: three translations and three rotations representing the overall deviation from the reference position. The cable force parameters continue to reflect the effects of the cables on the structural pose and force equilibrium. Construction state identification is thus extended from a cable-force-only solution to the joint optimization of pose and cable forces.

After the state identification objective is defined, the structural response under different parameter combinations must be calculated using a finite element platform. OpenSeesPy was used to construct the structural finite element model so that the geometric updating and force–response analysis remained consistent for the construction states. Nodal displacements, pose changes, and related responses were used to construct the objective function.

The main arch components were discretized using spatial nonlinear beam elements, and the sectional response was represented using the fiber section method. Each steel tube section was divided into fibers according to its actual geometry, and the Steel01 material model was assigned to the fibers to simulate the mechanical behavior of the steel. The cables were modeled using truss elements assigned a tension-only elastic material. This representation captures their unilateral behavior in tension while preventing compression, maintains consistency between the simulated and actual cable behavior during construction, and facilitates parameter updating during cable force optimization and state identification. The arch springing boundaries were simplified according to the actual stage-specific constraints: the three translational degrees of freedom were restrained, and the three rotational degrees of freedom were released, representing supports restrained in spatial position but relatively free to rotate. The overall finite element model and the actual structure at the construction site are shown in Figures 2 and 3, respectively.

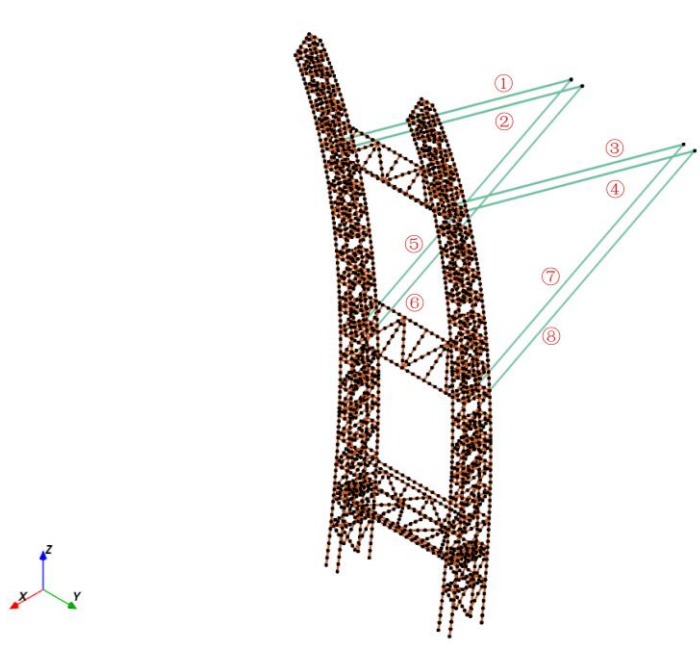


Figure 2 Finite element model and cable numbering



Figure 3 Actual structure at the construction site

3.3 Mathematical Optimization Model

In structural analysis, internal forces in cable elements alter the overall equilibrium state via geometric nonlinearity. For structural systems with mechanism-like behavior or substantial mobility, structural stiffness alone is often insufficient to restrain nodes near their initial modeled positions. Prestress may thus need to be applied to the cables to control the structural pose and nodal displacements.

3.3.1 Optimization Model for the Pre-Construction Reference State

The nodal coordinates, component connectivity, boundary conditions, and material parameters were determined from the design specifications. The initial forces in eight cables arranged in two layers were taken as the design variables. An optimal combination was sought such that the structure reached an equilibrium state under self-weight and cable forces that are as close as possible to the prescribed geometric position, thus minimizing both nodal displacements and overall pose deviation. This is a typical nonlinear parameter optimization problem whose basic form is as follows:

$$\min_{\mathbf{p}} J(\mathbf{p}) \quad (1)$$

where $\mathbf{p} = [T_1 \ T_2 \ \dots \ T_8]^T$ denotes the prescribed force parameters of the eight cables, whose numbering is shown in Figure 2, and $J(\mathbf{p})$ is the objective function measuring the structural deviation. Mechanical analysis gives the nodal displacement response at equilibrium as follows:

$$\mathbf{u}_i(\mathbf{p}) = [u_{ix}(\mathbf{p}) \ u_{iy}(\mathbf{p}) \ u_{iz}(\mathbf{p})]^T, \quad i = 1, 2, \dots, n \min J(\mathbf{p}) \quad (2)$$

where i is the node number and n is the total number of nodes included in the evaluation.

Cable force optimization seeks to return the overall structure as close as possible to its initial geometric position by adjusting the cable forces. The optimization criterion should thus minimize global nodal displacements. To prevent large displacements at individual nodes from being obscured by an averaged index, the objective function combines the sum of squared displacements at the constrained nodes with a penalty on the maximum nodal displacement:

$$J(\mathbf{p}) = \sum_{i \in \Omega} \|\mathbf{u}_i(\mathbf{p})\|^2 + \lambda_{\max} \max_{i \in \Omega} [\|\mathbf{u}_i(\mathbf{p})\|^2] \quad (3)$$

where Ω is the set of nodes included in the optimization and $\lambda_{\max}$ is the maximum displacement penalty coefficient. A larger $\lambda_{\max}$ drives the optimizer to reduce the largest nodal displacement, whereas a smaller value places greater emphasis on reducing displacement in an overall average sense.

3.3.2 Optimization Model for Construction Process States

When most monitoring data are unavailable during construction, the construction state can be identified from the reference-state model and a small number of key measurements. In this study, coordinates measured at nine key points during construction were used to identify the construction process states and provide a basis for subsequent force analysis and cable force identification.

With only nine measured coordinates, the selected control points should characterize the overall pose, local torsion, and displacement of key regions. They should be distributed at spatial locations sensitive to pose changes and be sufficient to constrain the principal modes of structural motion. To avoid imposing equal optimization constraints at all the measured points and thus compromising independent validation, we divided the nine key points into six constraint points and three validation points, as shown in Figure 4. The constraint points were used for structural registration and objective function construction and constrained the deviation between the current calculated and measured states. The validation points were excluded from optimization and used only to assess the consistency of the identified results. This division improves the independence of the assessment while maintaining identification accuracy, enabling full use of limited observations together with objective validation of the results.

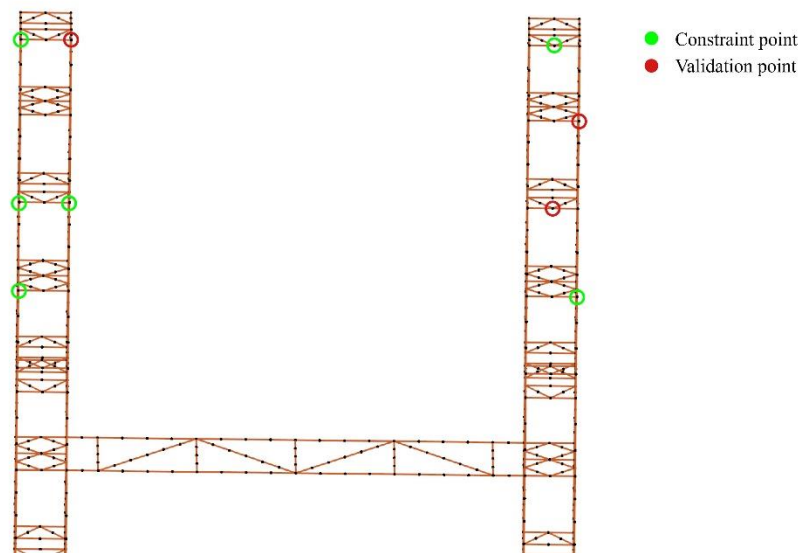


Figure 4 Division of model points into constraint and validation points

Because the overall spatial configuration at this stage can still be approximated as a rigid-body pose change, the measured points can first be registered to the corresponding initial model points using the least-squares method to obtain an approximate overall pose at the current instant [11]. Let the coordinates of the six constraint points in the initial model be $\mathbf{x}_i^{(0)} \in \mathbb{R}^3$, the measured coordinates be $\mathbf{x}_i^{(1)} \in \mathbb{R}^3$, and $i = 1, 2, \dots, 6$ denote the point number. The rigid-body transformation is as follows:

$$\hat{\mathbf{x}}_i^{(0)} = \mathbf{R}\mathbf{x}_i^{(0)} + \mathbf{t} \quad (4)$$

where $\mathbf{R} \in SO(3)$ is the rotation matrix and $\mathbf{t} = [t_x \ t_y \ t_z]^T$ is the translation vector. The corresponding least-squares registration problem is as follows:

$$\min_{\mathbf{R}, \mathbf{t}} \sum_{i=1}^6 \|\mathbf{R}\mathbf{x}_i^{(0)} + \mathbf{t} - \mathbf{x}_i^{(1)}\|^2 \quad \hat{\mathbf{x}}_i^{(0)} = \mathbf{R}\mathbf{x}_i^{(0)} + \mathbf{t} \quad (5)$$

subject to

$$\begin{cases} \mathbf{R}^T \mathbf{R} = \mathbf{I}, \\ \det(\mathbf{R}) = 1. \end{cases} \quad (6)$$

Here, $\mathbf{I}$ is the identity matrix.

When the data are severely incomplete, this step provides an approximate overall structural pose and establishes a reasonable initial configuration for subsequent state identification. The measured points, however, are not ideal rigid body points; their actual spatial positions are produced by the combined effects of structural self-weight, cable forces, temporary constraints, and stage-specific boundary conditions and thus include elastic deformation and force-induced offsets. The least-squares result can consequently serve only as an initial estimate before force analysis and cannot be used directly as the final mechanical analysis model.

Although the registered model is geometrically close to the measured points, its pre-loading pose does not fully reproduce the actual rotations and translations under construction conditions, producing errors in the geometric boundary conditions. If the rigid-body registration result is used directly in subsequent finite element analysis, discrepancies in nodal positions may be further amplified. Overall translational and rotational parameters must therefore be introduced in addition to the least-squares registration result and optimized jointly with the forces in the eight cables. Pose correction and cable force recalibration then produce a model that better represents the actual construction state.

The pre-loading model at this stage can be approximated as the pose obtained by superimposing rigid-body translations and rotations on the least-squares registration model. If $\mathbf{q}_i$ denotes the original coordinates of a point in the registered model, its coordinates $\mathbf{q}_i^*$ after three rotations and three translations are as follows:

$$\mathbf{q}_i^* = \mathbf{R}_x(\theta_x)\mathbf{R}_y(\theta_y)\mathbf{R}_z(\theta_z)\mathbf{q}_i + \mathbf{t}_e \quad (7)$$

where θ_x , θ_y , and θ_z are the rotations about the x , y , and z axes, respectively, and $\mathbf{t}_e = [\Delta x_e \ \Delta y_e \ \Delta z_e]^T$ is the translation vector. The rotations were applied first about the x -axis, then about the y -axis, and finally about the z -axis, providing a parameterized description of the pre-loading pose correction.

After the overall translations, rotations, and prescribed forces in the eight cables are introduced, the optimization vector is defined as follows:

$$\mathbf{p} = [\Delta x_e \ \Delta y_e \ \Delta z_e \ \Delta \theta_x \ \Delta \theta_y \ \Delta \theta_z \ T_1 \ T_2 \ \cdots \ T_8]^T \quad (8)$$

The first six variables describe the overall pre-loading pose correction, and T_i is the prescribed force to be identified for cable i . The overall pose obtained from least-squares registration is used as the initial state of the optimization problem to improve the solution efficiency and convergence stability.

Let $\mathbf{s}_i(\mathbf{p})$ be the simulated coordinates of constraint point i obtained from the finite element model under parameter vector $\mathbf{p}$. The objective function $J(\mathbf{p})$ is as follows:

$$\min_{\mathbf{p}} J(\mathbf{p}) = \sum_{i \in \Omega} \|\mathbf{s}_i(\mathbf{p}) - \mathbf{x}_i^{(1)}\|^2 + \lambda_{\max} \max_{i \in \Omega} [\|\mathbf{s}_i(\mathbf{p}) - \mathbf{x}_i^{(1)}\|^2] \quad (9)$$

3.4 Optimization Procedure

The objective function for the state identification problem is evaluated by finite element analysis. Because the structural response and optimization variables are nonlinearly coupled, analytical gradients are difficult to obtain, and the objective function may contain multiple local optima. A two-stage strategy combining global search and local optimization was therefore adopted. Differential evolution (DE) [12,13] was first used to search the parameter space globally and obtain a favorable initial solution. Sequential least squares programming (SLSQP) [14,15] was subsequently used for refined local optimization to improve identification accuracy and computational efficiency.

3.4.1 Global Search

During the global search stage, DE was used to optimize all the parameters. DE is a derivative-free optimization method based on a population search mechanism. Because it imposes few requirements on objective function continuity and differentiability, it is suitable for nonlinear optimization problems driven by finite element analysis. Candidate solutions are updated through population initialization, mutation, crossover, and selection. The algorithm searches for a favorable region over the entire parameter space, thus reducing the risk of convergence to a local optimum.

Let $\mathbf{X}_i^{(g)}$ denote individual i in generation g . A mutation vector is constructed using the classical differential mutation strategy:

$$\mathbf{V}_i^{(g)} = \mathbf{X}_{r_1}^{(g)} + k (\mathbf{X}_{r_2}^{(g)} - \mathbf{X}_{r_3}^{(g)}) \quad (10)$$

where $\mathbf{V}_i^{(g)}$ is the mutation vector; r_1 , r_2 , and r_3 are mutually distinct random integer indices that also differ from i ; and k is the scaling factor. During crossover, the mutation vector is combined with the current individual to form a trial vector $\mathbf{U}_i^{(g)}$. The binomial crossover is as follows:

$$u_{i,j}^{(g)} = \begin{cases} v_{i,j}^{(g)}, & j^* \leq c \text{ or } j = j_r, \\ x_{i,j}^{(g)}, & \text{otherwise,} \end{cases} \quad (11)$$

where $u_{i,j}^{(g)}$ and $v_{i,j}^{(g)}$ are the j th decision-variable values of $\mathbf{U}_i^{(g)}$ and $\mathbf{V}_i^{(g)}$, respectively; j^* is a random number in $[0,1]$; c is the crossover probability; and j_r is a random integer in $\{1, 2, \dots, D\}$, which ensures that at least one component of the trial vector is inherited from the mutation vector.

During selection, the fitness values of the trial vector and current individual are compared, and the vector with the better objective function value is retained for the next generation. If the objective function is denoted by $f(\cdot)$, the update rule is as follows:

$$\mathbf{X}_i^{(g+1)} = \begin{cases} \mathbf{U}_i^{(g)}, & f(\mathbf{U}_i^{(g)}) \leq f(\mathbf{X}_i^{(g)}), \\ \mathbf{X}_i^{(g)}, & f(\mathbf{U}_i^{(g)}) > f(\mathbf{X}_i^{(g)}). \end{cases} \quad (12)$$

Through this selection mechanism, the objective function values of the population decrease overall, and the algorithm progressively approaches the optimum. After multiple generations, DE provides a globally favorable parameter combination that serves as the initial solution for local optimization.

3.4.2 Local Optimization

DE provides a favorable initial solution, but its stochastic search characteristics leave room for further refinement near the final solution. SLSQP was therefore applied after the global search to refine the state parameters locally.

SLSQP uses local gradient information to rapidly determine the optimum under variable-bound constraints. Let the n -dimensional vector of variables to be identified be as follows:

$$\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_n]^T \quad (13)$$

and let the objective function be $\Gamma(\mathbf{x})$. The local optimization problem is

$$\min_{\mathbf{x}} \Gamma(\mathbf{x}) \quad (14)$$

subject to

$$\mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max} \quad (15)$$

where $\mathbf{x}_{\min}$ and $\mathbf{x}_{\max}$ are the lower and upper variable bounds, respectively. SLSQP uses the globally favorable solution obtained by DE as its initial value and iteratively updates the variables while satisfying the bound constraints until the change in the objective function or the variable increment meets the convergence criterion. Because the initial solution is already near the global optimum region, SLSQP further reduces the objective function value, improves the parameter identification accuracy, and decreases the computational cost.

4 Results and Discussion

4.1 Evaluation Method and Indices for Optimization Results

The validity of the state identification results should not be judged solely from the objective function value. Structural-response characteristics must also be considered to evaluate the optimization comprehensively. Analysis of the nodal-displacement distribution provides a more complete description of the overall force and deformation control performance under cable forces and supports assessment of the engineering feasibility of the optimized results.

Several error parameters were used to evaluate the structural control performance. Let Ω be the set of evaluated nodes and let $\mathbf{e}_i(\mathbf{p})$ be the nodal position error vector. The maximum nodal position error $e_{\max}$ is as follows:

$$e_{\max} = \max_{i \in \Omega} \|\mathbf{e}_i(\mathbf{p})\| \quad (16)$$

The value of $e_{\max}$ directly reflects deformation at the most unfavorable node. A small $e_{\max}$ indicates that displacement is effectively controlled even at the most unfavorable position, supporting overall structural safety and construction accuracy. The mean nodal position error $\tilde{e}$, which describes the overall deformation level, is defined as the arithmetic mean of the magnitudes of the position error vectors as follows:

$$\tilde{e} = \frac{1}{n} \sum_{i=1}^n \|\mathbf{e}_i(\mathbf{p})\| \quad (17)$$

The value of $\tilde{e}$ reflects the uniformity of the overall structural deformation. A small $\tilde{e}$ indicates that most nodes undergo only minor deformation and that overall control is satisfactory. The root-mean-square nodal position error e_{rms} provides an important supplementary measure as follows:

$$e_{\text{rms}} = \sqrt{\frac{1}{n} \sum_{i=1}^n \|\mathbf{e}_i(\mathbf{p})\|^2} \quad (18)$$

Compared with the mean error, e_{rms} is more sensitive to large displacements and highlights the influence of relatively large local deviations on overall performance. Because the objective function contains squared displacement terms, the root-mean-square displacement is strongly related to the objective function and can be used to assess the statistical consistency and validity of the optimization results.

4.2 State Identification Results

The state identification results comprise the pre-construction reference state and the construction process states. These two problems differ in terms of variable composition, objective function definition, and solution procedure. The identification of the reference state primarily concerns the prescribed forces in the eight cables. For the construction process states, mechanical parameters and six-degree-of-freedom pose-correction parameters are jointly optimized from the initial pose obtained by least-squares rigid-body registration. The optimization settings are listed in Table 2.

Table 2 Optimization settings for state identification

Parameter	Pre-construction reference state identification	Construction process state identification
Maximum number of DE iterations	40	35
DE population size	20	25
Nodes included in objective function	All nodes	Six constraint points
Objective function parameter λ_{max}	2.0	0.0
Prescribed strain range of prestressing-strand cables	[0, 0.005]	[-0.08, 0.0045]
Structural translation-parameter range/m	-	[-0.05, 0.05]
Structural rotation-parameter range/rad	-	[-0.03, 0.03]
Maximum number of SLSQP iterations	100	100
SLSQP accuracy threshold	1.0×10^{-8}	1.0×10^{-8}
SLSQP update step	1.0×10^{-6}	1.0×10^{-6}

On the basis of the state identification model and cable force optimization method described above, calculations were performed for the pre-construction reference state and three construction process states. The optimization results include the prescribed forces in the eight prestressing-strand cables arranged in two layers, the overall structural translation and rotation parameters, the objective function value $J(\mathbf{p})$, the maximum error e_{max} , the mean error $\bar{e}$, and the root-mean-square error e_{rms} .

4.2.1 Identification Results for the Pre-Construction Reference State

Table 3 summarizes the prescribed cable forces and the identified cable forces in the final equilibrium condition for the reference state, denoted as S0. The identified

forces fall within a reasonable range. Because the structural geometry at this stage was directly specified by the drawing-based model, the prescribed cable forces primarily establish the initial force equilibrium of the reference state, providing a clear physical meaning. The cable force distribution satisfies overall structural equilibrium and is consistent with the spatial arrangement of the cable system, demonstrating the stability of reference-state identification.

Table 3 Cable force identification results for the pre-construction reference state

No. of prestressing strand cable	Prescribed cable force /kN	Cable force at equilibrium /kN	Cable stress at equilibrium /MPa
1	8.18	9.41	17.69
2	8.18	9.40	17.67
3	8.18	9.40	17.67
4	8.18	9.41	17.69
5	5.89	0	0
6	5.89	0	0
7	5.89	0	0
8	5.89	0	0

The error statistics in Table 4 further demonstrate the high fitting accuracy of the pre-construction reference state. The objective function value is low, and the maximum, mean, and root-mean-square errors are all small. For a structure with a characteristic dimension of hundreds of meters, these errors indicate satisfactory model recovery and strong consistency between the identified reference state and the design geometry. Together, the objective function and error indices show that the cable force identification for the pre-construction reference state converged well numerically and satisfied the initial control requirements in engineering terms.

Table 4 Objective function values and error statistics for the pre-construction reference state

Optimization index	Objective function $J(\mathbf{p})$	Maximum error $e_{\max}$	Mean error $\bar{e}$	Root-mean-square error e_{rms}
Optimized result	0.55 m ²	0.029 m	0.014 m	0.015 m

4.2.2 Identification Results for Construction Process States

The three construction process states were discrete conditions extracted from a continuous construction sequence. Each was identified within the same joint pose and cable force framework. Unlike the pre-construction reference state, these states were not controlled solely by the cable force parameters. Starting from the initial pose obtained by least-squares rigid-body registration, three translation parameters and three rotation parameters were introduced to correct the overall spatial position, while the force parameters of the eight prestressing-strand cables were identified simultaneously. The resulting parameters characterize both spatial offsets and cable force redistribution during construction.

Table 5 presents the pose correction parameters for the three construction states, denoted as S1–S3. These parameters are incremental corrections relative to the corresponding least-squares registration results.

Table 5 Identified pose correction parameters for the construction process states

Construction state	Δx_e /m	Δy_e /m	Δz_e /m	$\Delta \theta_x$ /rad	$\Delta \theta_y$ /rad	$\Delta \theta_z$ /rad
S1	0.116	-0.020	3.54×10^{-3}	2.69×10^{-4}	1.25×10^{-3}	1.91×10^{-4}
S2	-0.033	0.038	0.024	3.60×10^{-4}	2.07×10^{-3}	9.40×10^{-5}
S3	0.014	0.083	-3.54×10^{-3}	-3.23×10^{-3}	-0.020	3.84×10^{-3}

Table 6 lists the final relative position parameters, which describe changes in the structural position between each optimized construction state and the pre-construction reference state. These parameters include the principal construction rotation θ_y , the axis deflection angles θ_x and θ_z , and the small translational displacements of the axis during construction, x_f , y_f , and z_f . The results show that pose changes relative to the initial state include not only increasing principal rotation but also spatial deflection and translation of the axis. Therefore, the structural motion did not fully conform to the restraints of an ideal fixed hinge, which is consistent with the actual condition. In addition to capturing relative changes among construction states, the results quantify axis offsets caused by incomplete restraint at the arch-springing hinges. This demonstrates the accuracy and interpretability of the method for identifying spatial pose changes during construction.

Table 6 Changes in position between the construction process and reference states

Construction state	θ_y /rad	θ_x /rad	θ_z /rad	x_f /m	y_f /m	z_f /m
S1	6.06×10^{-3}	-6.09×10^{-4}	-0.212	0.363	0.535	0.268
S2	9.87×10^{-3}	2.13×10^{-3}	-0.214	-0.797	-0.723	0.255
S3	0.402	1.60×10^{-3}	-0.210	-0.580	-0.849	0.107

Table 7 summarizes the equilibrium forces T_c in the eight cables under gravity after pose correction for the three construction states. The value of T_c represents the actual force distribution under the combined effects of gravity and pose correction under the corresponding construction conditions, and σ_c represents the actual stress in each cable. Although the cable forces increased markedly in the later construction stage and the material stress reached 50.78% of the specified tensile strength of 1860 MPa, the overall force level remained essentially controllable and did not exceed the load-carrying capacity.

Table 7 Cable force identification results for the construction process states

Cable No.	S1: T_c /kN	S1: σ_c /MPa	S2: T_c /kN	S2: σ_c /MPa	S3: T_c /kN	S3: σ_c /MPa
1	0	0	0	0	502.46	944.47
2	0	0	0	0	496.11	932.54
3	0	0	0	0	454.25	853.85
4	0	0	0	0	0	0
5	142.37	267.61	86.04	161.73	485.58	912.74
6	25.18	47.33	139.04	261.35	466.40	876.69
7	0	0	0	0	434.98	817.63
8	0	0	0	0	0	0

The internal force and stress responses at the arch springings are listed in Table 8. The structural force level was relatively stable in the pre-construction reference state, with comparatively low axial forces, bending moments, and stresses at the arch springings. The internal forces increased moderately during the first two construction states, and the arch springings remained within a stable force range. In the later state, the axial compressive force and bending moment increased markedly, accompanied by a corresponding increase in stress. Although the maximum stress increased substantially at this stage, it remained below the material yield stress of 420 MPa. The arch-springing forces therefore remained controllable throughout construction, and no state exceeded the material load-carrying capacity.

Table 8 Internal force and stress responses at the arch springings

Construction state	Maximum axial compressive force N_c /kN	Maximum bending moment M_c /(kN·m)	Maximum stress σ_c /MPa
S0	1,661.69	41.83	47.27
S1	1,725.00	45.35	49.28
S2	1,810.41	53.77	52.37
S3	3,331.99	206.51	107.76

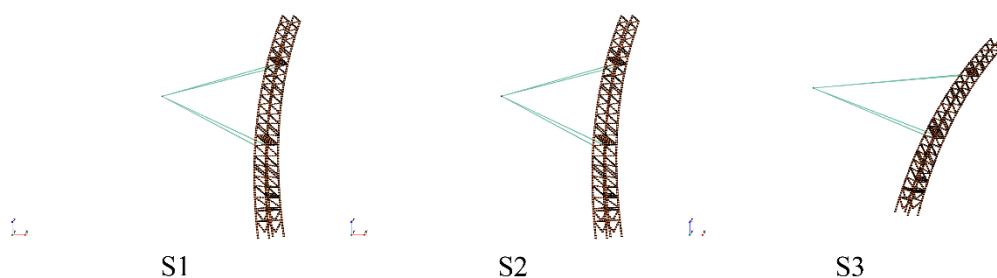
The objective function values and error statistics for the three construction states are presented in Table 9. The objective function values range from 10^{-4} m² to 10^{-3} m², indicating satisfactory convergence. The maximum, mean, and root-mean-square errors at the constraint and validation points are essentially of the same order of magnitude and exhibit similar trends. The joint identification results are thus consistent for both the points included in the optimization and the independent validation points. Among the three states, S1 produced the smallest errors, whereas S3 produced relatively larger errors; nevertheless, all the errors remained low.

Table 9 State identification indices for the construction process states

State	Objective function $J(p)$ /m ²	$e_{\max}$, constraint points /m	$e_{\max}$, validation points /m	$\bar{e}$, constraint points /m	$\bar{e}$, validation points /m	e_{rms} , constraint points /m	e_{rms} , validation points /m
S1	3.89×10^{-5}	3.42×10^{-3}	3.47×10^{-3}	2.30×10^{-3}	1.96×10^{-3}	2.55×10^{-3}	2.24×10^{-3}
S2	3.02×10^{-5}	9.81×10^{-3}	7.47×10^{-3}	6.61×10^{-3}	6.00×10^{-3}	6.14×10^{-3}	7.09×10^{-3}
S3	4.12×10^{-4}	1.22×10^{-2}	1.05×10^{-2}	7.61×10^{-3}	9.16×10^{-3}	8.28×10^{-3}	9.29×10^{-3}

4.2.3 Comparison of Structural Poses during Construction

The structural poses of the construction process states are compared in Figure 5. The geometric profiles and nodal positions deviate to varying degrees among the states.



a) Elevation views of the identified construction process states

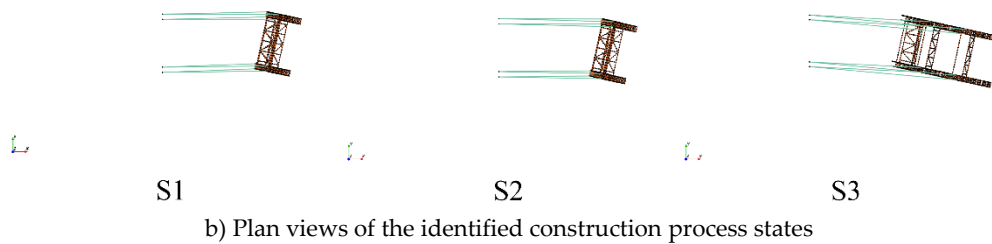


Figure 5 Structural poses during construction

As shown in Figure 5, the overall geometry progressively deviated from the designed construction position as the construction advanced, whereas spatial coordinate offsets gradually emerged at key locations. The identified model states agree with the actual construction states, indicating that the proposed method effectively captures the evolution of the structural pose during construction. The results reveal overall and directional spatial offsets as construction progresses, demonstrating that construction-stage changes are strongly identifiable. They further show that a state identification model based on only a few measured points can recover the principal spatial configuration at each stage and characterize the progressive evolution of the structural pose, providing a reliable basis for subsequent state assessment and geometric control.

5 Conclusions

A structural state identification method based on limited monitoring information was developed to address incomplete observations of construction-stage states, scarcity of monitoring data, and simultaneous changes in structural pose and force conditions. The method was verified using the vertical rotation construction of the main arch of an overpass bridge. The principal conclusions are as follows.

- (1) A parameterized construction-state identification method based on limited observations was developed. Complex construction states were abstracted into a finite-dimensional state vector. The prescribed cable forces were the principal variables for the pre-construction reference state. For construction process states, coordinates measured at a small number of key points were used as observational constraints, and structural pose-correction parameters and cable force parameters were introduced after least-squares rigid-body registration. The geometric configuration and key force-related parameters of the construction state were thus described and identified within a unified framework.
- (2) A framework for the joint identification of structural poses and cable forces was established. Because changes in nodal positions during construction are influenced by both overall spatial pose and force-state variations, translations in three directions, rotations about three axes, and cable forces were jointly identified using finite element analysis and a global–local optimization strategy. This approach avoids describing the construction state solely through geometric changes or a single set of cable force parameters. The engineering case demonstrates that the method can identify pose changes, including the principal construction rotation, axis deflection, and spatial translation, and can reproduce the stage-dependent redistribution of the cable forces.
- (3) The effectiveness of state identification under limited observations was verified. Only a few key measured points were used to constrain each construction process state, and independent validation points were retained to assess the results. For the three construction states, the errors at the constraint and validation points were of similar magnitude. The identified states reproduced the progressive spatial pose changes during construction, demonstrating that a

small number of representative observations can effectively constrain the principal construction-stage state parameters.

- (4) An engineering case study demonstrates that the proposed method provides an identification approach based on limited observations and finite element modeling for construction-stage state assessment, spatial geometric control, and key parameter updating when monitoring information is incomplete. Its principal feature is the integration of structural pose changes and force conditions within a unified parameter identification framework, providing a reference for inferring the overall state of a complex bridge under construction from a small amount of observational information.

Conflicts of Interest: The authors declare no conflicts of interest.

Data Availability Statement: The data supporting the findings of this study are available from the corresponding author, Yuxiao Wang, upon reasonable request.

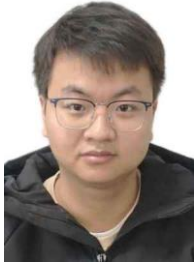
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